ESTIMATING THE MEAN BLOOD FLOW of ARM
BASED ON WINDKESSEL MODEL

Shing-Hong Liu¹, Da-Chuan Cheng² and Jia-Jung Wang³

¹ Department of Computer Science and Information Engineering, Chaoyang
University of Technology, Taichung, 41349, Taiwan, R.O.C.

² Department of Biomedical Imaging and Radiological Science, China Medical
University, Taichung, Taiwan, R.O.C.

³ Department of Biomedical Engineering, I-Shou University, Kaoshiung, Taiwan, ROC

E-mail: shliu@cyut.edu.tw (S-H Liu);

Corresponding author: Shing-Hong Liu.
Tel: 886 + (0)4 + 2332-3000 ext 5211
FAX: 886 + (0)4 + 23329898
Abstract

The blood flow is always used to evaluate the arterial obstruction or arteriosclerosis. The echo-method is the standard. In this study, we propose a new method for assessing the total mean blood flow of the arm using a known brachial arterial compliance and based on a two-element Windkessel (WK) model, in which the arterial hemodynamics consists of a peripheral arterial resistor and a peripheral arterial capacitor in parallel. The known compliance of the brachial artery belonging to a local and relative amount has been estimated from the pattern of the oscillometric waveform in our previous study. An estimating peripheral arterial compliance and resistance are get from the known compliance. The governing equation of the two-element WK model is then used to get the blood flow waveform. The accuracy of WK-based mean blood flow ($F_{WK}$) is validated by comparing it with the mean blood flow ($F_{echo}$) estimated by an echo-based method in the brachial arteries of 32 subjects. The results showed that the values of $F_{WK}$ and $F_{echo}$ were significantly correlated ($r = 0.671$). This suggests that a total mean blood flow can be evaluated by the known arterial compliances derived by regional or relative changed measurements.

Keywords: Windkessel, Oscillometry, Blood Flow
1. Introduction

Although, the blood flow and pressure are normally used to detect arterial characteristics, the profile of the arterial blood flow could be more used as a potential marker of generalized arteriosclerotic disease 1-3. The forearm blood flow also could be used to response relationships for different drugs or mediators 4. There were some studies of skeleton muscle workloads to measure blood flow. Resting brachial arterial flow, including retrograde flow, increases during lower limb exercise 5-7. In recent years, the endothelial vasodilator function has been widely studied which evaluation is to measure the blood flow and pressure 8. In these studies, they all used the echo-method to detect the blood velocity and the diameter in an artery of interest, and then the arterial blood flow is obtained by multiplying the measured velocity by the cross-sectional area 9-10.

However, if the echo-method is excluded from the blood flow measurement, most noninvasive methods focus on the change in the vascular volume. In electrical-impedance plethysmography, the blood flow is implicitly assessed by measuring impedance changes for electrical current passing through the arterial lumen11. Some studies have used the differential of the oscillometric waveform or the photoplethysmogram waveform to estimate the brachial blood flow 12-13. In laser Doppler sonography, the arterial blood flow is indirectly derived to detect the velocity and amount of red blood cells in an arterial vessel 14.

When blood propagates from the heart to the peripheral vascular system, pressure and flow waveforms are affected by the properties of the cardiovascular system. Therefore, there were many studies using analog models of the arterial system to understand vascular physiology. Peripheral resistance and peripheral arterial compliance form the major part of the arterial load on the heart15-16. Frank first introduced the two-element Windkessel (WK) model who considered the whole
arterial tree as an elastic chamber with a constant compliance and a resistance representing the total resistance of arterial tree\textsuperscript{17}. Many models for the systemic arterial compliance consist of a circuit with lumped parameters\textsuperscript{18-20}. In these models, the peripheral resistance was calculated by dividing the mean arterial pressure by the mean blood flow. During diastole there is no blood inflow from the heart, and so the peripheral arterial compliance can be found from the decay time of the pressure waveform and the peripheral resistance. Stergiopulos evaluated the many different WK models to estimate the peripheral arterial compliance, and found that method based on the two-element WK model was more accurate than those based on the three-element WK models\textsuperscript{21}.

The goal of this study utilizes a known brachial arterial compliance and pressure waveform to detect the total mean blood flow of the arm based on a WK model. The known brachial arterial compliance was detected by oscillometric method which belongs to a local and relative changed amount\textsuperscript{22}. According to the WK model, the peripheral arterial compliance of the arm not only includes the brachial arterial compliance, but also includes the radial and ulnar arterial compliances and other arteriolar compliances. However, the brachial artery is the major artery in these arteries. The first step is to estimate the total peripheral arterial compliance from the known brachial arterial compliance. The second step is to use the estimating peripheral arterial compliance and a full cycled pressure waveform which was extracted from the oscillometric waveform when the cuff pressure deflated to the diastolic blood pressure to estimate the peripheral arterial resistance. Finally, the WK-based blood flow waveform was constructed by the governing equation of the two-element WK model. The mean blood flows of 32 patients whose diameter and flow velocity of the brachial artery were measured by echo-based method were used as a standard to compare the WK-based mean blood flow for evaluating the accuracy
of our proposed method.

This paper is organized as follows. Section 2 detail describes how to estimate the parameters of WK model with the known brachial arterial compliance. In section 3, the total blood flow waveform of the arm is made, and its mean blood flow is compared with the echo-based mean blood flow. The results are discussed and conclusions are drawn in Section 4.

2. Materials and Methods

In the previous study, we had designed a measurement system to record the cuff pressure and flow signal during the inflation and deflation periods. The brachial arterial compliance ($C_{ast}$) under the different loaded condition has a detail descriptions in Appendix A. In this section, we briefly describe a two-element WK model and hemodynamic characteristics of 32 subjects. Then, a calibrated transfer function was made to estimate the peripheral arterial compliance with the known brachial arterial compliance. According to WK model, the estimated peripheral arterial resistance was calculated with the estimating peripheral arterial compliance.

2.1 Two-element Windkessel Model

In the two-element WK model, the whole arterial tree is modeled as an elastic chamber with a peripheral arterial compliance, $PAC$, and a peripheral arterial resistance, $PAR$. The governing equation of the two-element WK is

$$PAC \frac{dP(t)}{dt} + \frac{P(t)}{PAR} = F(t),$$

where $F$ denotes the blood flow and $P$ is the blood pressure. In the frequency domain, the corresponding input impedance is given by
During diastole, there is no blood inflow from the heart ($F=0$), and hence the right-hand side of Eq. (1) vanishes and a direct integration yields

$$P(t) = P(t_1) e^{-(t-t_1)/\tau PAR PAC},$$

where $P(t_1)$ is the initial blood pressure in the diastolic phase. Equation (3) expresses a monoexponential decay and can be fitted to any portion of the diastolic waveform to yield the time constant, $\tau$:

$$\tau = PAR \cdot PAC.$$

2.2 Clinical measurement

32 subjects (14 men and 18 women) aged 68 ± 15 years (mean ± SD; range: 19 to 93 years) who had been explained the study protocol before the measurement were measured the brachial blood pressure of the left arm with an electric sphygmanometer (DINAMAP PROCARE 100, GE Medical Systems, USA) at Hsinchu Hospital. The brachial arterial internal diameter and velocity of blood flow of each subject was measured by the duplex ultrasound scan (7.5MHz, M2410B, AGILENT, Hewlett Packard). The hemodynamic characteristics of 32 subjects were shown in Table 1 that include the brachial arterial compliances of three different loaded conditions. The different loaded condition is that the cuff around the brachial artery of interest is deflated to be the mean arterial pressure (MAP), systolic blood pressure (SBP) or diastolic blood pressure (DBP). Thus, the different loaded brachial arterial compliances are $C_{osc}(MAP)$, $C_{osc}(SBP)$ and $C_{osc}(DBP)$ respectively.

2.3 Estimation of Peripheral Arterial Compliance

The arterial resistance of periphery was calculated by dividing the mean blood
pressure by the mean blood flow. The blood flow was obtained by multiplying the measured mean velocity by the cross-sectional area. Thus, the mean echo-based flow ($F_{echo}$) was defined using the following equation:

$$F_{echo} = \pi \cdot D^2 \cdot \int_0^T \frac{v(t)dt}{4T},$$

(5)

where $D$ is the arterial internal diameter, and $T$ is the cardiac cycle, and $v$ the blood flow velocity. The range of $F_{echo}$ is from 0.189 to 2.693 ml/second (0.954 ± 0.547 ml/second).

According to the definition of $PAR^{21}$, the range of $PAR$ is from 29.7 to 539.7 mm Hg · second/ml (136.3 ± 99.6 mm Hg · second/ml). A full cycled waveform was extracted from an oscillometric waveform for each subject when the cuff pressure deflated to the DBP. The waveform was calibrated to a pressure waveform by the SBP value and DBP value of the subject. Equation (3) was applied to the diastolic phase of pressure waveform to determine time constant ($\tau$) whose range is from 0.37 to 1.81 second (0.912 ± 0.348 second). The range of the regression coefficient ($R$) is from 0.973 to 0.999 (0.993 ± 0.006).

Figure 1(a) shows the blood pressure waveform extracted from the oscillometric waveform when the mean cuff pressure was deflated to DBP, and calibrated by the blood pressure, for one subject (sex: female; age: 44 years; SBP: 112 mmHg; DBP: 62 mmHg; MAP: 83 mmHg). Figure 1(b) shows the diastolic phase of the blood pressure waveform which was fitted with Eq. (3) to produce a regression coefficient of 0.992 and the following parameter values: time constant ($\tau$), 1.033 second. Her mean echo-based flow is 0.56 ml/second. So, the $PAR$ is 149.3 mm Hg · second/ml.

The $PAC$ was calculated by Eq. (4) whose range is from 0.0007 to 0.0338 ml/mm Hg (0.0105 ± 0.0091 ml/mm Hg). Statistics of WK model’s parameters for 32 subjects were in Table 2. The linear regression between the $PAC$ and three
known loaded brachial arterial compliance, \( C_{osc}(MAP) \), \( C_{osc}(SBP) \) and \( C_{osc}(DBP) \), is shown in Table 3. We could find that \( PAC \) and \( C_{osc}(DBP) \) has a best correlation coefficient, \( r = 0.783 \), as shown in Fig. 2. The calibrated transfer function is below:

\[
PAC = 14.32C_{osc}(DBP) - 4.42.
\]  
Therefore, the known loaded brachial arterial compliance could be changed to the range of \( PAC \) by this transfer function. We called this estimated compliance as the estimated peripheral arterial compliance (EPAC) whose range was from 0.0012 to 0.128 ml/mm Hg (0.0143 ± 0.0211 ml/mm Hg).

2.4 Estimation of Peripheral Arterial Resistance

In previous section, the \( EPAC \) has been got from the known brachial arterial compliance by the calibrated transfer function. According to Eq. (4), we used the time constant and \( EPAC \) to calculate the estimated peripheral arterial resistance (EPAR) whose range was from 36.7 to 433.4 mm Hg·second/ml (123.7 ± 90.8 mm Hg·second/ml). We also used a Bland–Altman\(^\text{23}\) plot for the \( PAR \) detected from the echo-based method and the \( EPAR \) detected from the \( EPAC \) of 32 subjects to provide an agreement figure, as shown in Fig. 3.

3. Results

In section 2, we have presented how to determine \( EPAR \) and \( EPAC \) from the known brachial arterial compliance. The blood pressure waveform extracted from the oscillometric waveform when the mean cuff pressure was deflated to DBP was calibrated using the SBP and DBP. Then, according to Eq. (1), the total blood flow waveform of the arm could be made by \( EPAR \), \( EPAC \), and the blood pressure waveform. The WK-based mean blood flow, \( F_{WK} \), was calculated with a full cycled
flow waveform. Figure 4 shows an constructed blood flow waveform for the same subject as in Fig. 1 whose peak $F_{WK}$ is 1.47 ml/second, and average $F_{WK}$ is 0.32 ml/second. Figure 5 shows the velocity and diameter of the subject which were measured by the echo method. The peak velocity is 57.3 cm/second, and the diameter is 0.24 cm. Thus, the peak $F_{echo}$ is 2.62 ml/second, and the average $F_{echo}$ is 0.56 ml/second. To validate the proposed method, the $F_{WK}$ was compared with the echo-based mean blood flow ($F_{echo}$), as shown in Fig. 6. The correlation coefficient ($r$) between $F_{echo}$ and $F_{WK}$ values is 0.671. These results indicated that with the local and relative changed compliance, the mean blood flow of the arm obtained using the WK model was in moderate agreement with the echo-based blood flow.

4. Discussions

Now, the image methods have been used in clinical diagnosis to measure the absolute change of the arterial diameter and evaluate the blood flow, including ultrasound and magnetic resonance imaging (MRI)\textsuperscript{3, 25}. But, the limited spatial resolution of the high-frequency ultrasound often makes it difficult to determine the arterial diameter (and hence the flow) precisely\textsuperscript{3, 25}. The arterial blood flow also can be evaluated by electrical-impedance plethysmographic and photoplethysmographic methods. One of the major drawbacks of electrical-impedance plethysmography is that its signal is extremely sensitive to motion artifacts and interference from electrical power lines\textsuperscript{11}. Similarly, photoplethysmographic signals are vulnerable to body motion and system instability\textsuperscript{12-13}. Moreover, these two methods require a transfer function to transform the plethysmographic signal into a flow-related signal.

Long-term changes in arterial blood flow induce parallel changes in arterial size, with increased flow stimulating outward remodeling and decreased flow reducing the arterial size. Recent studies have shown that the brachial blood flow is elevated in
patients with risk factors and cardiovascular disease\textsuperscript{26, 27}. In these clinical studies, the baseline brachial blood flow was calculated using the flow velocities as measured ultrasonically and based on the vessel cross-sectional area. It typically takes 5–10 minutes to measure these parameters, which makes it difficult to apply in home-based care. However, in our studies we have used an airflow meter and pressure transducer to construct the loaded compliance-vs-pressure curve based on oscillometry*. The two-element WK model could be used to estimate the $F_{WK}$. Because our proposed is based on oscillometry, the measurement can be made in only 30–40 sec.

The WK model has been accepted as appropriate for describing the arterial tree, and suitable for obtaining the $P_{AC}$\textsuperscript{18-20}. The most well-known method is the diastolic decay method, in which a monoexponential curve is fitted to the decay time of the pressure waveform, and the $P_{AR}$ is commonly derived from the ratio between the MAP and flow. Thus, the $P_{AC}$ is a total compliance of the arm which includes the major arterial compliances, brachial, radial, and ulnar arterial compliances, and minor compliances, arteriolar and capillary compliances. However, in our previous study, the loaded brachial arterial compliance just only considers the brachial artery, and is a regional and relative changed measurement. But, in Table 3, we could find that $C_{osc}(DBP)$ has a best correlation with the $P_{AC}$, $r = 0.783$. This result represents that the $C_{osc}$ can be used to evaluate the $P_{AC}$. Using $P_{AC}$ and $C_{osc}(DBP)$ data, a calibrated transform function was made.

In section 2.4, the $E_{PAR}$ was evaluated from the $E_{PAC}$ with Eq. (4), and $E_{PAC}$ was transferred from $C_{osc}$ with Eq. (6). But, $P_{AR}$ was evaluated from the measured parameters, $F_{echo}$ and MAP. Moreover, $E_{PAR}$ and $E_{PAC}$ is a nonlinear relation. A smaller error in $E_{PAC}$, a larger error in $E_{PAR}$. That is why the scale of the different error in Fig. 3 is very large and the correlation coefficient between $P_{AR}$ and $E_{PAR}$ is
very close to 0. We validated the proposed $F_{WK}$ by comparing it with $F_{echo}$. The values of $F_{echo}$ in this study were calculated from the vessel cross-sectional area and the mean flow velocity (in cm/second). However, there are several potential sources of errors in the echo-based method. The first possible error source is the limited resolution of the echo system in measuring the internal radius of the artery, as expressed in Eq. (5). Second, the cross section of the brachial artery is here assumed to be constant, and ignoring vessel contraction or dilatation in this way will unquestionably introduce errors into the blood flow computation. Third, the distribution of blood flow velocity in the direction perpendicular to the arterial axis is nonuniform, which introduces inaccuracy into echo-based velocity measurements. Finally, the echo-based blood flow is an total and absolute measurement. However, the $C_{osci}$ is only a regional and relative measurement. The above factors increase the errors of the mean blood flow in the comparison between the two methods. Therefore, if we used another gold standard method to measure the blood flow, like as MRI, the proposed WK-based method will be reinforced its reliability.

The second term on the left-hand side of Eq. (1) is the reciprocal of arterial resistance multiplying the blood pressure. Because the mean peripheral resistance was 123.7 second mm Hg/ml and the mean blood pressure was 92.3 mm Hg, this term will be very small. Therefore, the first term will be the dominant factor for estimating the WK-based flow waveform. That is, a more-accurate value for the compliance will increase the accuracy of the $F_{WK}$. Although blood only flows during the systolic phase of the heart, $C_{osci}(DBP)$ is more accurate than $C_{osci}(SBP)$ in Table 3. Thus, we used $C_{osci}(DBP)$ to evaluate $F_{WK}$.

5. Conclusions

In the present study, although $r$ between $F_{echo}$ and $F_{WK}$ is only 0.671, we first used
a known arterial compliance to estimate the arterial blood flow based on the WK model. The arterial compliance could also be measured using many different non-echo methods which commonly belong to the indirect measurements or the regional and relative changed measurements, such as the pulse wave velocity\textsuperscript{28}, impedance plethysmography\textsuperscript{11}, photoplethysmography\textsuperscript{12}, oscillometry\textsuperscript{22,29}, and model-based approaches\textsuperscript{30}. Therefore, these methods could be further applied to estimate the total mean blood flow in combination with the technique presented here. As our discussion, the more accurate at compliance, the more accurate at blood flow.

These results indicate that our proposed method based on the two-element WK model can be readily used to obtain a mean brachial arterial flow without requiring echo-based measurements. Since the method for computing the WK-based mean blood flow using the loaded brachial arterial compliance can be suitably implemented in an oscillometric blood pressure monitor, it has great potential in home-care applications.

**Appendix A: Loaded Arterial Compliance**

A pumped airflow and cuff pressure sensors were simultaneously recorded during the inflating period, with the airflow converted into the volume by integration. When commencing measurements, we assumed that the cuff had been wound tightly around the upper arm, meaning that the cuff pressure was zero and the residual air inside the cuff could be ignored. The cuff and body tissue are elastic, and the elastic stretch of the cuff rubber bladder will be a nearly linear elastic properties in the high range of cuff pressure. Therefore, an exponential function was used for the cuff model:

\[
V_c = V_{C0} + a(1 - e^{-bP_{C,I}^2}) ,
\]

where \(P_{C,I}\) is the air pressure inside the cuff during the inflating period, \(V_{C0}\) is the
initial air volume in the cuff, and $V_C$ is the volume of air pumped to the cuff. Therefore, the slope of the cuff model, $C_{cuff}$, is

$$C_{cuff}(P_{c-1}) = \frac{dV_c}{dP_{c-1}}. \quad A2$$

During the deflating period, cuff pressure ($P_{C,D}$) and oscillometric amplitude (OA) were used as the input and output variables to describe the envelope of oscillation waveform. According to the oscillometry, OAs could be considered as the cuff pulse pressure since they resulted from the change in the arterial volume ($V_{pulse\_artery}$). Therefore, the change in the arterial volume embedded in the cuff volume can be calculated from $C_{cuff}$ and OAs:

$$V_{pulse\_artery}(P_{C,D}) = C_{cuff}(P_{C,D})OA(P_{C,D}). \quad A3$$

The change in the arterial volume is defined here as the instantaneous change in the vascular area in the arterial volume segment under the cuff. The transmural pressure is defined as

$$P_t = \bar{P}_a - \bar{P}_c. \quad A4$$

where $\bar{P}_a$ is the mean arterial pressure, $\bar{P}_c$ is the mean cuff pressure. The arterial dynamic compliance has been consistently defined as:

$$C(P_t) = \frac{\Delta V(P_t)}{\Delta P_{pulse}}. \quad A5$$

where $\Delta V$ is the arterial volume change, and $\Delta P_{pulse}$ is the arterial pulse pressure. In our studies, it took about 40-50 seconds to perform a steady oscillometric pressure measurement. During this period, it is acceptable that the MAP is assumed to constant. Under that assumption, the linear relation is present between $P_t$ and $\bar{P}_{C,D}$. Thus, the loaded compliance of brachial artery under the different cuff pressures is defined as the following equation,
\[ C_{osc}(Pc_D) = \frac{V_{pulse\_artery}(Pc_D)}{\Delta P_{pulse}}. \]  \hspace{1cm} (A6)

According to this formula, \( C_{osc}(Pc_D) \) could be considered to be a function of the cuff pressure, and be used to elucidate the properties of the arterial compliance.

**Acknowledgment**

This work was supported by the National Science Council, Taiwan, Republic of China, under grant numbers NSC 99-2221-E-324 -001.
References


Table 1 Baseline hemodynamic characteristics of 32 subjects

| Internal diameter of brachial artery (cm) | 0.344 ± 0.079 |
| Mean Brachial blood flow velocity (cm/second) | 47.7 ± 13.8 |
| MAP (mm Hg) | 92.3 ± 11.9 |
| SBP (mm Hg) | 135.4 ± 23.6 |
| DBP (mm Hg) | 67.5 ± 8.8 |

Data are mean±SD values. MAP is mean arterial pressure, SBP is systolic blood pressure, DBP is diastolic blood pressure.

Table 2 Statistics of WK model’s parameters for 32 subjects.

| Mean Brachial blood flow (ml/second) | 0.954 ± 0.547 |
| Peripheral arterial resistance (mm Hg · second/ml) | 136.3 ± 99.6 |
| Peripheral arterial compliance (ml/mm Hg) | 0.0105 ± 0.0091 |
| Time constant (second) | 0.912 ± 0.348 |

Table 3 The total arterial compliance was compared with three different loaded brachial arterial compliances for 32 subjects.

<table>
<thead>
<tr>
<th></th>
<th>PAC vs. ( C_{asci(MAP)} )</th>
<th>PAC vs. ( C_{asci(SBP)} )</th>
<th>PAC vs. ( C_{asci(DBP)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r )</td>
<td>0.604</td>
<td>0.524</td>
<td>0.783</td>
</tr>
<tr>
<td>SEE</td>
<td>0.0004</td>
<td>0.0004</td>
<td>0.0003</td>
</tr>
</tbody>
</table>

\( PAC \) is the peripheral arterial compliance, \( r \) represents a correlation coefficient, SEE represents a standard error of estimate.
Fig. 1. (a) The blood pressure waveform for one patient whose age is 44 years, SBP is 112 mmHg, DBP is 62 mmHg and MAP is 83 mmHg, (b) the diastolic waveform (dash line) was fitted with a monoexponential decay function (solid line) : $P(t) = P(t_1)e^{-\left(t-t_1\right)/RC}$, where $P(t_1)=112$, $RC=0.968$, $R=0.992$. 
Fig. 2 Scatter plot of $C_{osc}(DBP)$ vs. $PAC$: $n = 32$, $r = 0.783$ ($p < 0.0001$), SEE = 0.0003. Linear function: $y = 14.32x - 4.42$. 
Fig. 3 A Bland–Altman plot of each PAR and EPAR for 32 subjects.
Fig. 4. The evaluated blood flow waveform for the same subject as in Fig. 1.
Fig. 5. The velocity and diameter were measured by the echo method for the same subject as in Fig. 1
Fig. 6. Scatter plot of $F_{WK}$ vs. $F_{echo}$: $n = 32$, $r = 0.671 \ (p < 0.0001)$, SEE = 0.0003, $y = 0.289 + 0.75x$. 